Ridge Regression

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Math 243: Stat Learning

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Outline

In today's class, we will. . .

- Discuss Ridge Regression as an alternate method of model selection
- Implement Ridge Regression using the `glmnet` package

Section 1

Penalized Regression

Motivation 1

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When might this new model have lower MSE than the original model?

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  - How might the variance and bias of the following model compare to the standarized model?

$$\hat{y} = 10 + 0.02z_2 + 500z_1$$

Ridge Regression

Recall that least squares regression estimates $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_p$ for

$$\hat{y} = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \epsilon$$

are obtained by finding the values of β that minimize

$$\mathrm{RSS} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2$$

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- With a shrinkage penalty, the algorithm prefers models with lower coefficients.
- This tends to reduce variance, at the cost of increased bias.

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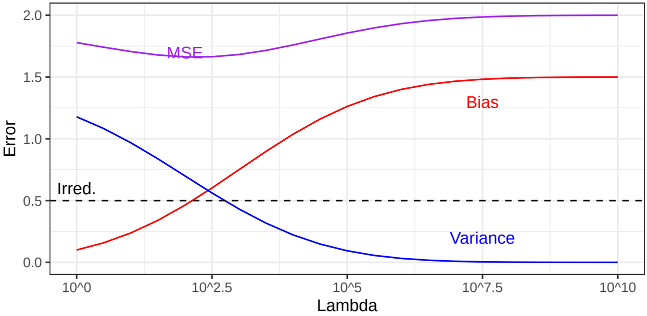

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Ridge regression is most efficient if predictors are standardized first.

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{\hat{\sigma}_j}$$

Where x_{ij} is the ith observation of the jth predictor, $\bar{x}_j$ is the sample mean of the jth predictor, and $\hat{\sigma}_j$ is the sample st. dev. of the jth predictor.

Section 2

Ridge Regression in R

The Data

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## # A tibble: 6 x 8
##   Name     Global_Sales NA_Sales EU_Sales JP_Sales Critic_Score User_Score Rating
##   <chr>           <dbl>    <dbl>    <dbl>    <dbl>        <dbl>      <dbl> <chr>
## 1 Rayman~          0.07     0.06     0.02    0               75        7.9 E
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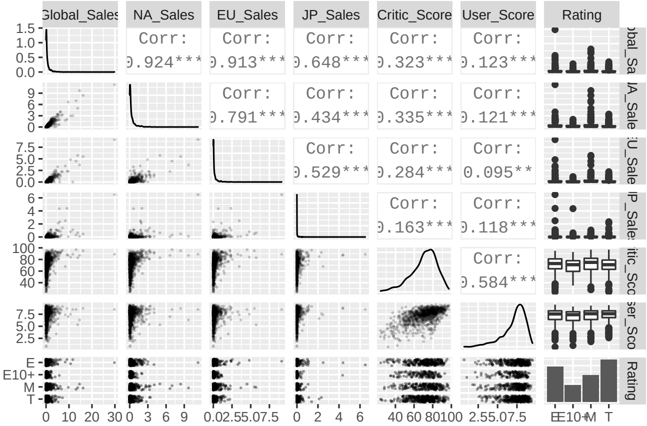
- What are some possible problems with the full model?

Exploration



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We also create vector `grid` of suitable tuning parameters λ.

```
## [1] 10000000000  7564633276  5722367659  4328761281  3274549163  2477076356
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- By default, glmnet standardizes observations. To use unstandardized obs. add standardize = FALSE
- Here, we gave a specific range of values for the tuning parameter. But if no lambda value is supplied, the function will automatically select a range.

Manipulating output of glmnet

```
coef(ridge_mod)[,1:4]
```

```
## 9 x 4 sparse Matrix of class "dgCMatrix"
##                         s0            s1            s2            s3
## (Intercept)   8.162200e-01  8.162200e-01  8.162200e-01  8.162200e-01
## NA_Sales      3.412480e-10  4.511098e-10  5.963407e-10  7.883273e-10
## EU_Sales      4.792437e-10  6.335319e-10  8.374919e-10  1.107115e-09
## JP_Sales      5.824358e-10  7.699458e-10  1.017823e-09  1.345502e-09
## Critic_Score  7.209905e-12  9.531071e-12  1.259951e-11  1.665582e-11
## User_Score    2.582969e-11  3.414533e-11  4.513812e-11  5.966994e-11
## RatingE10+   -5.247474e-11 -6.936852e-11 -9.170109e-11 -1.212235e-10
## RatingM       7.685481e-11  1.015975e-10  1.343060e-10  1.775446e-10
## RatingT      -6.427287e-11 -8.496495e-11 -1.123187e-10 -1.484787e-10
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```

```
ridge_mod$lambda[75]
```

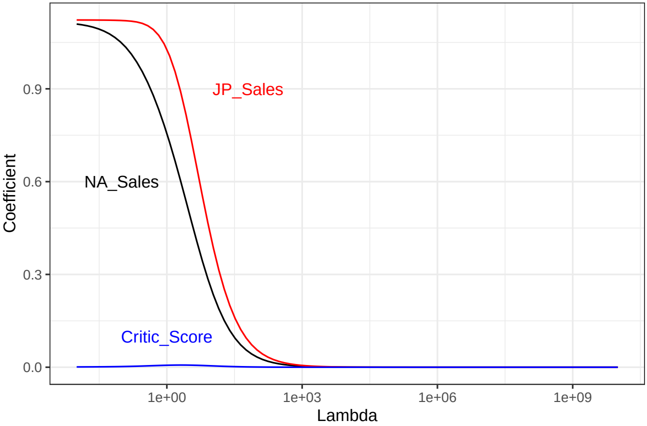
```
## [1] 10.72267
```

```
coef(ridge_mod)[,75]
```

```
##  (Intercept)     NA_Sales     EU_Sales     JP_Sales Critic_Score   User_Score
##  0.239041734  0.234144568  0.325084954  0.386330686  0.004219627  0.012114496
##   RatingE10+      RatingM      RatingT
## -0.030652286  0.042868156 -0.037871886
```

The effects of ridge regression

```
dd<-data.frame(b_NA_Sales<- coef(ridge_mod)[2,],
               b_JP_Sales<- coef(ridge_mod)[4,],
               b_Critic = coef(ridge_mod)[5,],
               grid)
```



Optimizing the tuning parameter

Which performs better, the model with $\lambda \approx 10000$ or $\lambda \approx 10$?

```
set.seed(11)
video_games_trn<-video_games %>% sample_frac(.75)
video_games_tst<-anti_join(video_games, video_games_trn)

x_trn<-model.matrix(Global_Sales ~. - Name, data = video_games_trn)[,-1]
y_trn<-video_games_trn$Global_Sales

x_tst<-model.matrix(Global_Sales ~. - Name, data = video_games_tst)[,-1]
y_tst<-video_games_tst$Global_Sales

ridge_mod_trn <- glmnet(x_trn, y_trn, alpha = 0, lambda = grid)

pred_10 <- predict(ridge_mod_trn, s = 75, newx = x_tst)
pred_10k <- predict(ridge_mod_trn, s = 50, newx = x_tst)
```

Optimizing the tuning parameter

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```
set.seed(11)
video_games_trn<-video_games %>% sample_frac(.75)
video_games_tst<-anti_join(video_games, video_games_trn)

x_trn<-model.matrix(Global_Sales ~. - Name, data = video_games_trn)[,-1]
y_trn<-video_games_trn$Global_Sales

x_tst<-model.matrix(Global_Sales ~. - Name, data = video_games_tst)[,-1]
y_tst<-video_games_tst$Global_Sales

ridge_mod_trn <- glmnet(x_trn, y_trn, alpha = 0, lambda = grid)

pred_10 <- predict(ridge_mod_trn, s = 75, newx = x_tst)
pred_10k <- predict(ridge_mod_trn, s = 50, newx = x_tst)
```

```
MSE_10<-mean( (y_tst - pred_10)^2 )
MSE_10k<-mean( (y_tst - pred_10k)^2 )
data.frame(MSE_10, MSE_10k)
```

```
##     MSE_10  MSE_10k
## 1 1.364439 1.291877
```

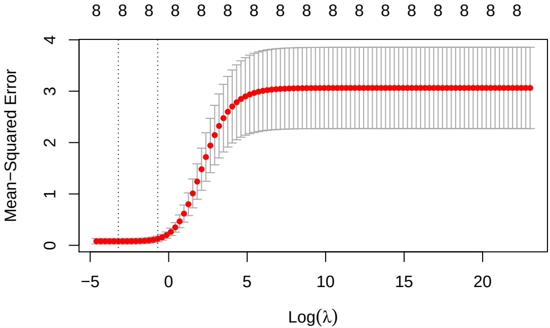
Optimizing the tuning parameter, cont'd

We can use the `cv.glmnet` function to perform cross-validation to compare MSE across all values of λ

Optimizing the tuning parameter, cont'd

We can use the `cv.glmnet` function to perform cross-validation to compare MSE across all values of λ

```
set.seed(1010)
my_cv<-cv.glmnet(x, y, alpha = 0, lambda = grid)
plot(my_cv)
```




```
best_L<-my_cv$lambda.min
best_L
```

```
## [1] 0.04037017
```