Regression and Classification Trees

Nate Wells

Math 243: Stat Learning

November 4th, 2020

Outline

In today's class, we will. . .

- Investigate pruning algorithms for improving accuracy of regression trees
- Discuss classifcation trees for classification problems.

Section 1

Improving Regression Trees

Trees on Trees

We use a subset of the `pdxTrees` dataset from the `pdxTrees` repo (maintined by K. McConville, I. Caldwell, and N. Horton)

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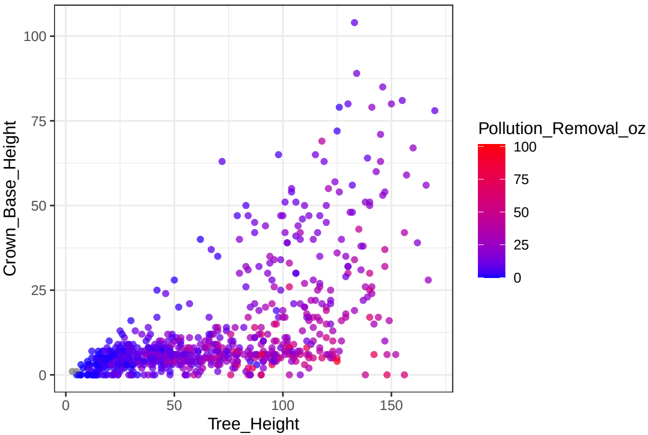
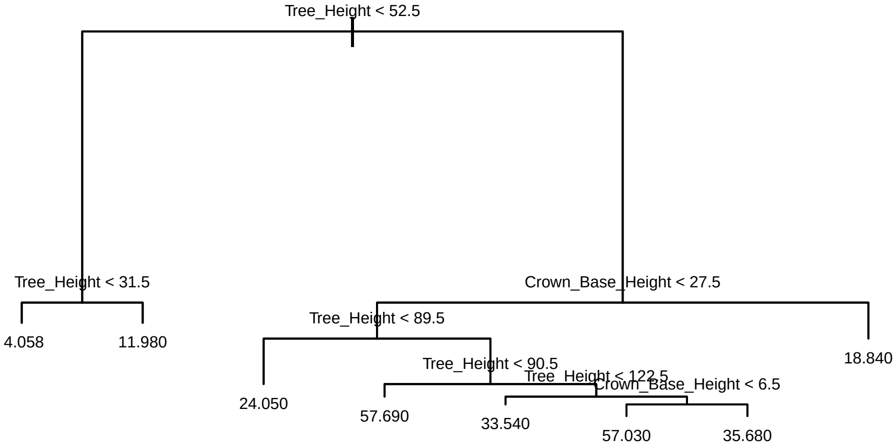
```
## Rows: 1,000
## Columns: 10
## $ Species              <fct> PSME, CAJA, QUMU, CADE, PSME, CPSP, PRAV, PSME...
## $ Condition            <fct> Fair, Fair, Fair, Fair, Fair, Fair, Poor, Fair...
## $ Tree_Height          <int> 102, 23, 18, 78, 123, 85, 11, 145, 16, 72, 88,...
## $ Crown_Width_NS       <int> 52, 36, 6, 17, 52, 36, 9, 36, 10, 86, 25, 12, ...
## $ Crown_Width_EW       <int> 43, 40, 6, 18, 38, 52, 11, 35, 10, 86, 10, 16,...
## $ Crown_Base_Height    <int> 63, 5, 5, 6, 13, 5, 6, 9, 5, 8, 6, 4, 4, 3, 2,...
## $ Structural_Value     <dbl> 6694.04, 2444.75, 71.28, 4162.43, 6159.02, 113...
## $ Carbon_Storage_lb    <dbl> 1992.9, 917.5, 5.3, 1428.7, 1901.4, 11071.6, 2...
## $ Stormwater_ft        <dbl> 78.9, 43.9, 1.0, 19.8, 117.6, 52.0, 4.1, 80.1,...
## $ Pollution_Removal_oz <dbl> 21.2, 11.8, 0.3, 5.3, 31.6, 14.0, 1.1, 21.5, 1...
```

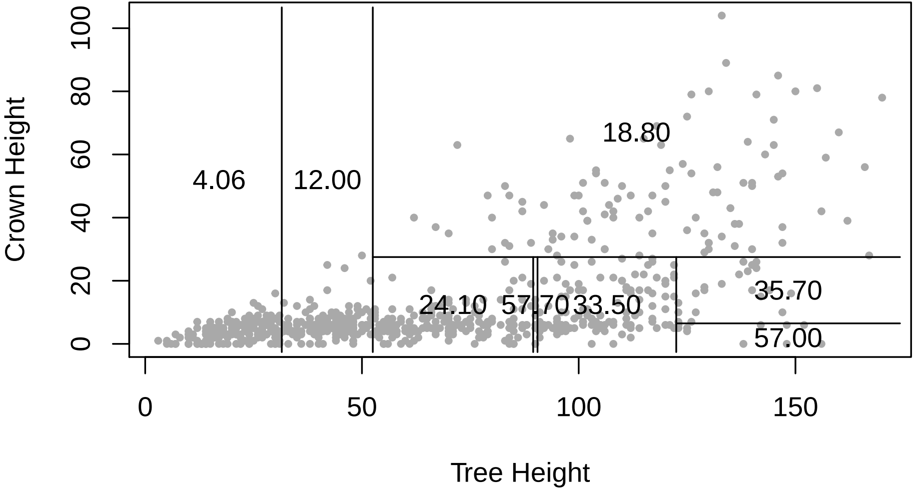
Pollution Removal



Regression Tree



Another Visualization



Tree Accuracy

Let's check MSE on a test set:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

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## 1 169.0145
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Nevertheless, what are some downsides to the tree model?

The general tree algorithm

1. Begin with the entire data set S and search every value of every predictor to cut S into two groups S_1 and S_2 that minimizes sum of squred error:

$$\text{SSE} = \sum_{i \in S_1} (y_i - \bar{y}_1)^2 + \sum_{i \in S_2} (y_i - \bar{y}_2)^2$$

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Consider the RSS of a **big** tree. How might training and test RSS compare?

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- But this search is actually even more computationally expensive than best subset!
- So we instead restrict our attention to those subtrees most likely to improve RSS

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$$\mathrm{RSS} + \alpha|T|$$

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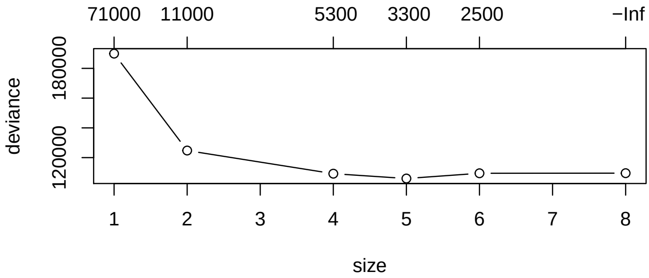
There are two ways to select the **best** subtree.

1. Choose the tree with smallest MSE.
2. Choose the *smallest* tree with MSE within 1 standard deviation of smallest MSE

Pruning Example

How does MSE vary as tree size changes?

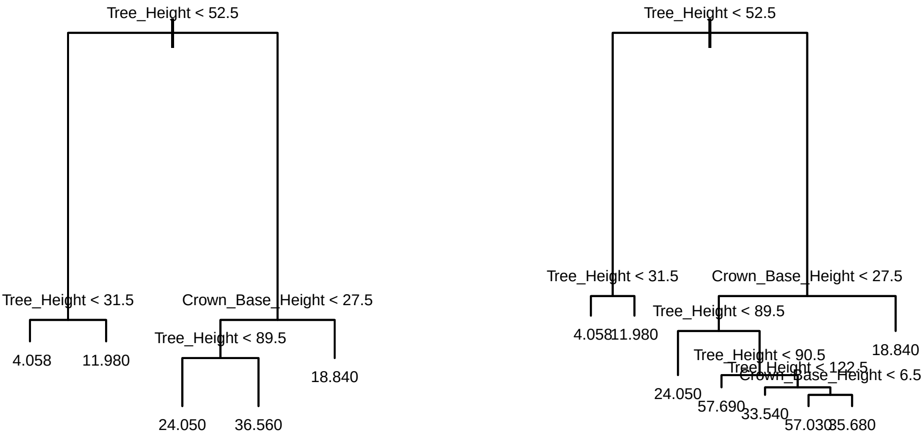


What are the test MSEs for the full tree and the subtree with 5 terminal nodes?

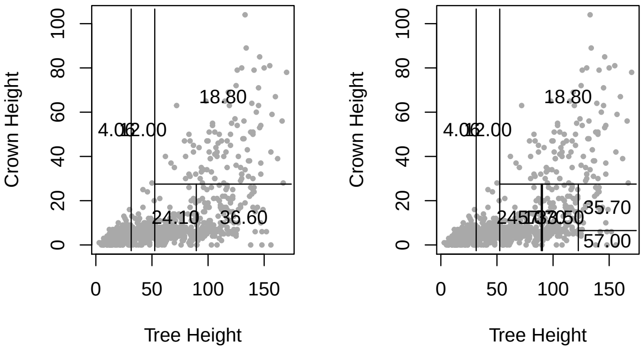
```
##   Full_Tree_MSE
## 1      169.0145
```

```
##   small_Tree_MSE
## 1       152.5175
```

Comparison


Tree_Height < 52.5
Tree_Height < 31.5
Crown_Base_Height < 27.5
Tree_Height < 89.5
4.058
11.980
18.840
24.050
36.560
Tree_Height < 52.5
Tree_Height < 31.5
Crown_Base_Height < 27.5
Tree_Height < 89.5
4.058
11.980
18.840
Tree_Height < 90.5
Tree_Height < 122.5
Crown_Base_Height < 6.5
24.050
57.690
33.540
57.030
35.680

Comparison 2



Creating Tree Models in R

There are two common packages for creating regression trees in R: `tree` and `rpart`.

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- The `tree` package is one of the oldest packages on CRAN. It is a (tiny) bit easier to use. But its plots are ugly. ISLR uses `tree`.
- The `rpart` package is newer, computationally faster, and has more options. It also can be combined with the `partykit` and `ggparty` packages for **much** nicer plots. Applied Predictive Modeling uses `rpart` along with `caret` for cv.

Trees using tree

To fit a tree:

```
library(tree)
tree_model<-tree(Pollution_Removal_oz ~ ., data = small_pdxTrees)
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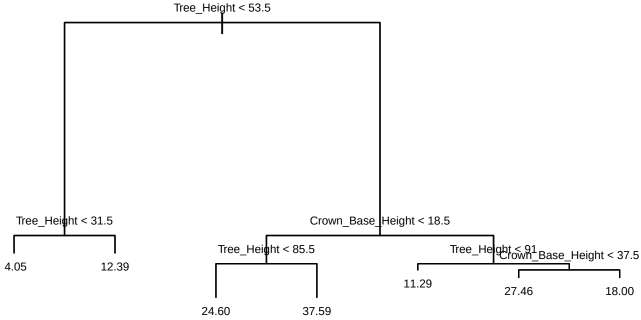
To view:

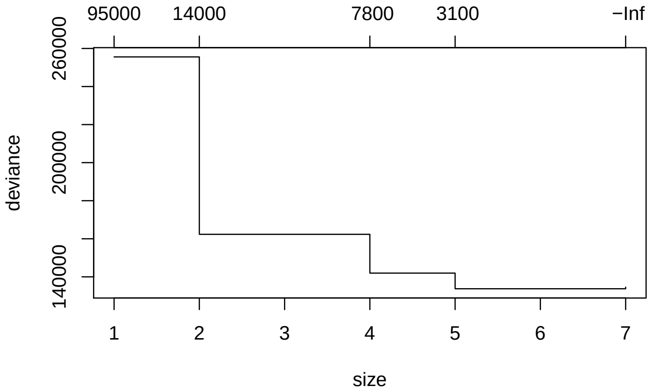
```
plot(tree_model)
text(tree_model, pretty = 0, cex = .5)
```



Trees in R via tree cont'd

To perform cost-complexity pruning:

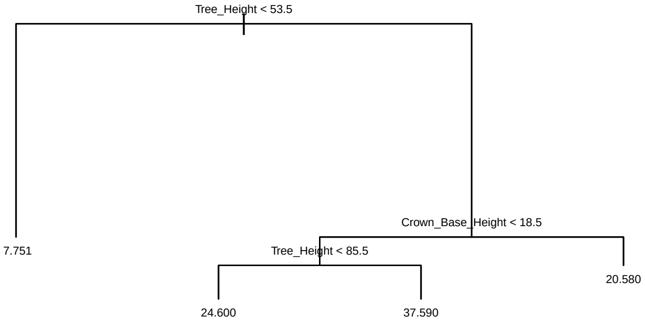
```
tree_model_cv<-cv.tree(tree_model)
plot(tree_model_cv)
```



Trees in R via tree cont'd

And to get a pruned tree:

```
pruned_tree<-prune.tree(tree_model, best = 4)
plot(pruned_tree)
text(pruned_tree, pretty = 0, cex = .5)
```



Section 2

Classification Trees

Trees for Classification Problems

Can we predict the winner of a presidential election based on demographics, state polling, economic conditions, and other features?

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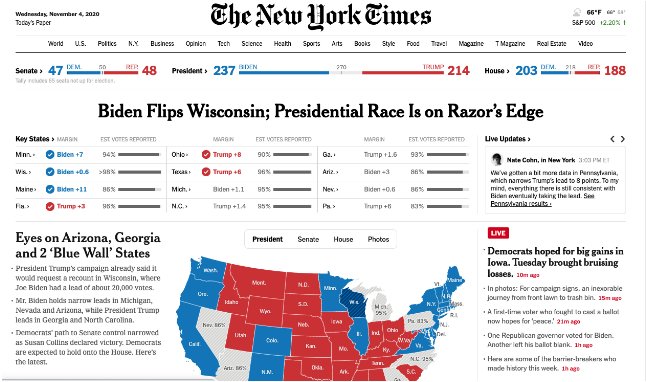
- No. Too stressful.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World U.S. Politics N.Y. Business Opinion Tech Science Health Sports Arts Books Style Food Travel Magazine T Magazine Real Estate Video

Senate › 47 DEM. 50 REP. 48
Tally includes 65 seats not up for election.
President › 237 BIDEN 270 TRUMP 214
House › 203 DEM. 218 REP. 188

Biden Flips Wisconsin; Presidential Race Is on Razor's Edge

Key States ›	Margin	Est. votes reported		Margin	Est. votes reported		Margin	Est. votes reported
Minn. ›	Biden +7	94%	Ohio ›	Trump +8	90%	Ga. ›	Trump +1.6	93%
Wis. ›	Biden +0.6	>98%	Texas ›	Trump +6	96%	Ariz. ›	Biden +3	86%
Maine ›	Biden +11	86%	Mich. ›	Biden +1.1	95%	Nev. ›	Biden +0.6	86%
Fla. ›	Trump +3	96%	N.C. ›	Trump +1.4	95%	Pa. ›	Trump +6	83%

Live Updates ›

Nate Cohn, in New York 3:03 PM ET
We've gotten a bit more data in Pennsylvania, which narrows Trump's lead to 8 points. To my mind, everything there is still consistent with Biden eventually taking the lead. See Pennsylvania results ›

Eyes on Arizona, Georgia and 2 'Blue Wall' States

- President Trump's campaign already said it would request a recount in Wisconsin, where Joe Biden had a lead of about 20,000 votes.
- Mr. Biden holds narrow leads in Michigan, Nevada and Arizona, while President Trump leads in Georgia and North Carolina.
- Democrats' path to Senate control narrowed as Susan Collins declared victory. Democrats are expected to hold onto the House. Here's the latest.

President Senate House Photos

LIVE

- Democrats hoped for big gains in Iowa. Tuesday brought bruising losses. 10m ago
- In photos: For campaign signs, an inexorable journey from front lawn to trash bin. 15m ago
- A first-time voter who fought to cast a ballot now hopes for 'peace.' 21m ago
- One Republican governor voted for Biden. Another left his ballot blank. 1h ago
- Here are some of the barrier-breakers who made history this week. 1h ago

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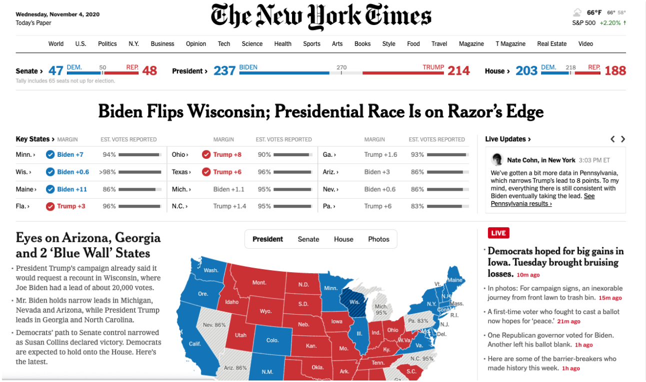

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- The Gini index is small if all $\hat{p}_{mk}$ are close to 0 or 1.