Multiple Linear Regression

Nate Wells

Math 243: Stat Learning

September 14th, 2020

Outline

In today's class, we will. . .

- Generalize the simple regression model to include more than 1 predictor
- Quantify model accuracy for linear regression models (both simple and multiple)
- Implement multiple regression in R

Section 1

Multiple Regression

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Could we get better predictive power by including all explanatory variables in the *same* model?

Multiple Regression Model

In a **simple linear regression model** (SLR), we express the response variable Y as a linear function f of one predictor variable X:

$$Y = f(X) + \epsilon$$

and estimate f using

$$\hat{Y} = \hat{f}(X) = \hat{\beta}_0 + \hat{\beta}_1 X$$

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- In the MLR model, we allow predictors to either be quantitative or binary categorical (i.e taking values 0 or 1 corresponding to failure or success)

Finding Parameters

To create an SLR model, we found the equation of a line that minimizes RSS, where

$$\mathrm{RSS} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_1),$$

which has the solution

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

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To create an MLR model. . .

we do the exact same thing!

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We even use the exact same R code to fit the linear model:

```
my_mod<-lm(Y ~ X1 + X2 + ... + Xp, data = my_data)
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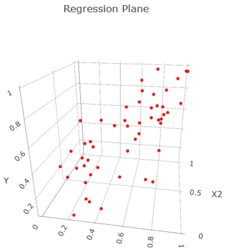
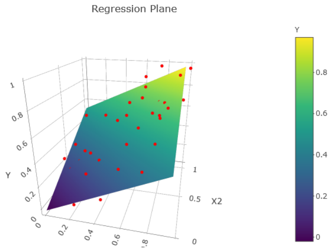
The Plane of Best Fit





An interactive graphic available under Monday 9-14 on schedule page on course website

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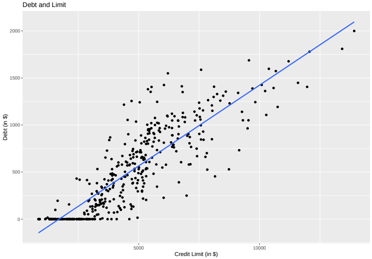

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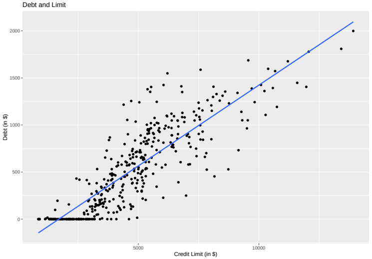

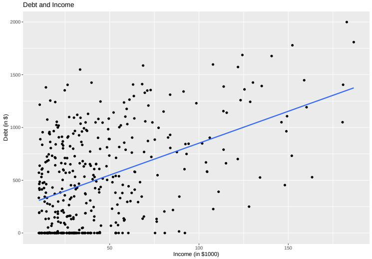

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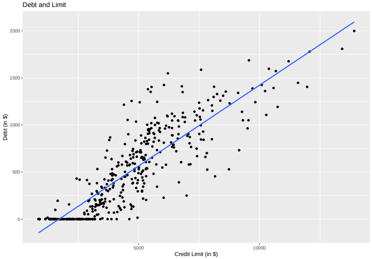

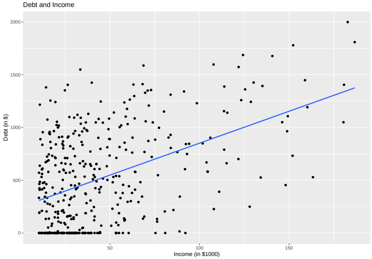

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Both variables have some explanatory power for Debt.

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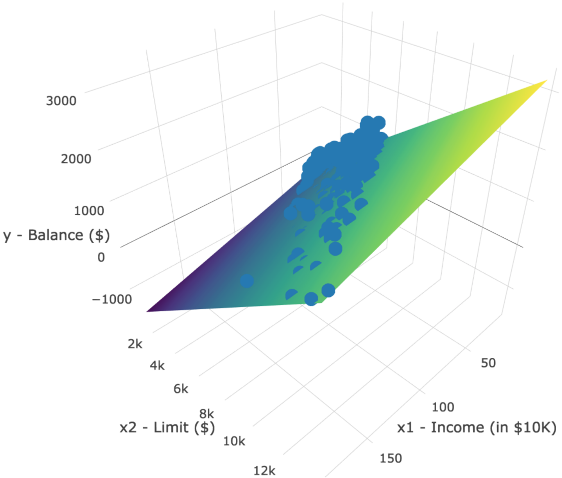

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- While for **fixed** value of Limit, increasing Income by \$1000 decreases debt by an average of \$7.66.

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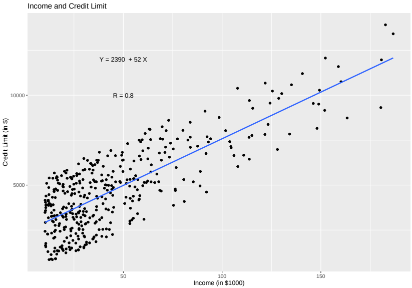

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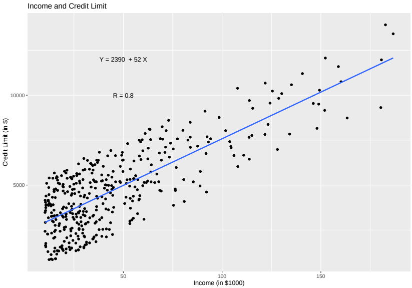

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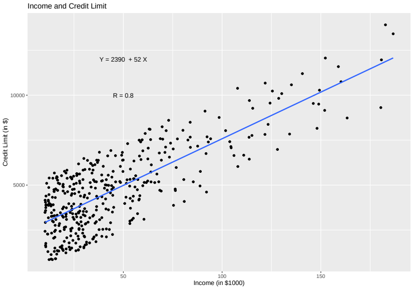

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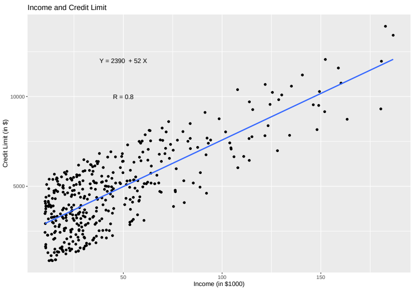

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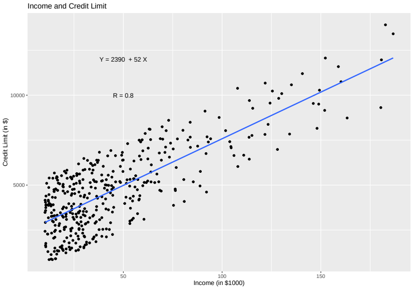

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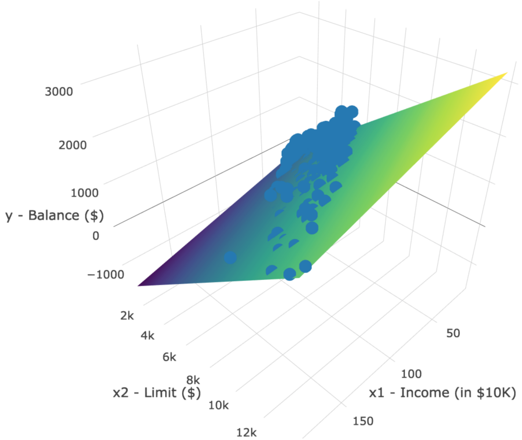

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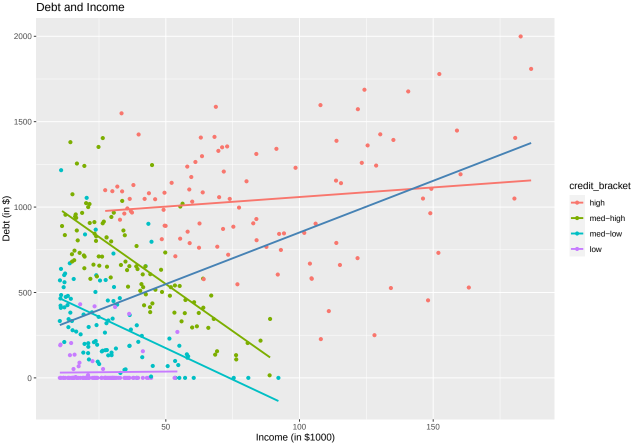

Section 2

Assessing Model Accuracy

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It has the property that

$$E(\mathrm{RSE}) \approx \mathrm{Var}(\epsilon)$$

Poll

Which of the following is most likely to decrease as more and more predictors are added to a linear model?

a. MSE
b. RSS
c. RSE
d. $\text{Var}(\epsilon)$

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- The value of R^2 is always between 0 and 1, and represents the percentage of variability in values of the response just due to variability in the predictors.

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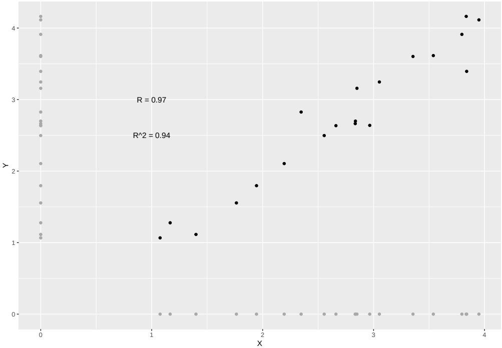

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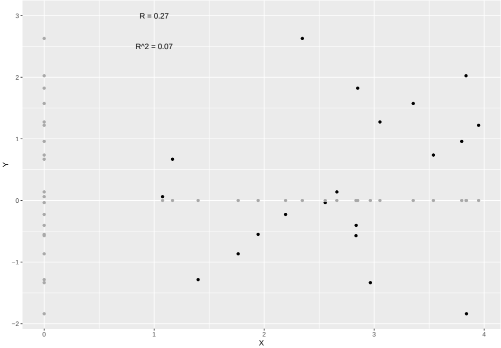

Formulas for R^2 in terms of correlation

For SLR,

$$R^2 = [\mathrm{Cor}(X, Y)]^2 = \left[\frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}\right]^2 = \left[\frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}\right]^2$$

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We will usually use software to compute R^2.

Model Accuracy in R

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mod_credit<-lm(Balance ~ Income + Limit , data = Credit)

summary(mod_credit)
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##
## Call:
## lm(formula = Balance ~ Income + Limit, data = Credit)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -232.79 -115.45  -48.20   53.36  549.77
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -385.17926   19.46480  -19.79   <2e-16 ***
## Income        -7.66332    0.38507  -19.90   <2e-16 ***
## Limit          0.26432    0.00588   44.95   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 165.5 on 397 degrees of freedom
## Multiple R-squared:  0.8711, Adjusted R-squared:  0.8705
## F-statistic:  1342 on 2 and 397 DF,  p-value: < 2.2e-16
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We can use `summary(mod)$r.sq` or `summary(mod)$sigma` to access R^2 and RSE directly.

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- This adjusted R^2 is usually a bit smaller than R^2, and the difference decreases as n gets large.

Testing Significance

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Why would it be incorrect to conduct p many significant tests comparing each predictor to the response?

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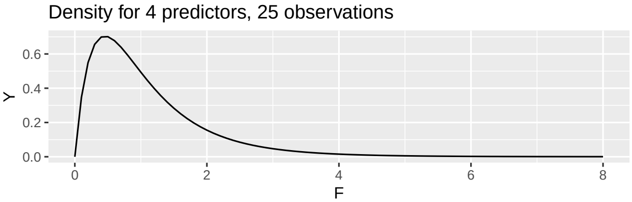

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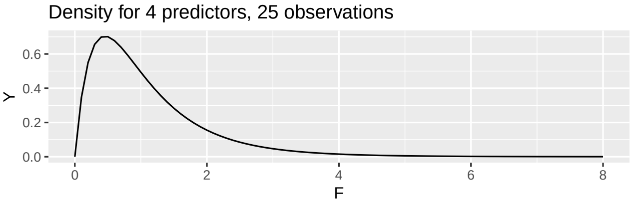

If H_0 is true, then F is typically close to 1, and rarely much greater.