MLR: Troubleshooting

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Math 243: Stat Learning

September 21st, 2020

Outline

In today's class, we will. . .

- Troubleshoot potential problems with the linear model

Section 1

Problems with Linear Model

Overview

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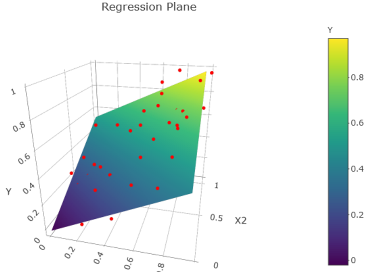

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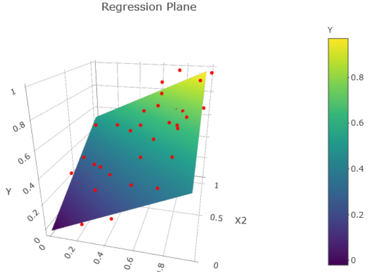

However, if we want to make *predictions* or perform *statistical inference* we need to make sure key assumptions of randomness are met.

Common Problems

Most problems fall into 1 of 6 categories:

1. Non-linearity of relationship between predictors and response
2. Correlation of error terms
3. Non-constant variance in error
4. Outliers
5. High-leverage points
6. Collinearity of predictors

Non-linearity

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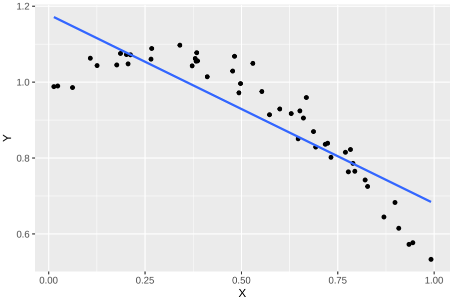

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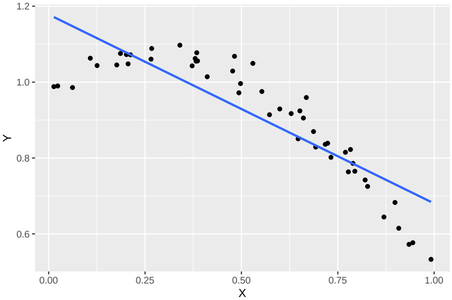
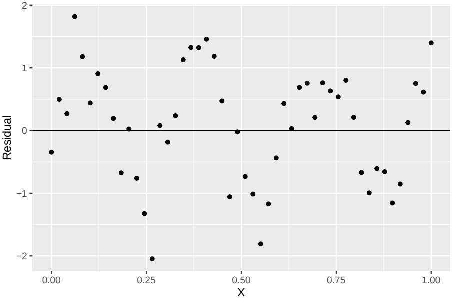


But if this assumption is false, our model is likely to have high bias.

Correlation of Errors

If errors are correlated, then knowing the values of one gives extra information about values of others.



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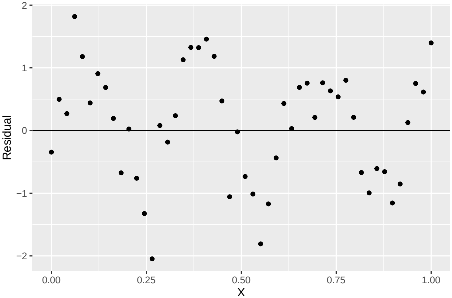
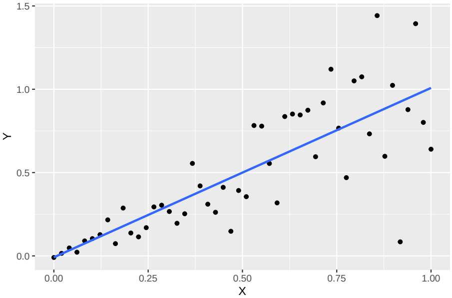


Correlated errors lead to underestimates of residual standard error - Producing narrower confidence intervals and inflating test statistics

Non-constant variance

For prediction and inference with LM, we assume that all residuals have the same variance.



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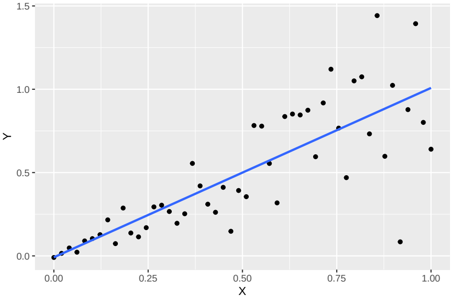
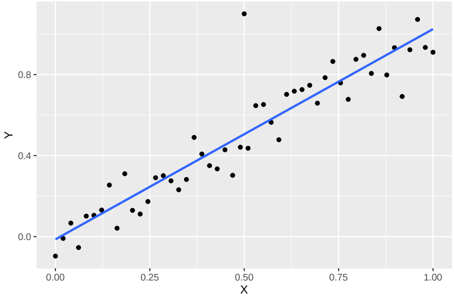


Least squares regression does not minimize RSS; requires more data for accurate predictions

Outliers

While outliers may occur even if model assumptions are met, they do influence accuracy estimates



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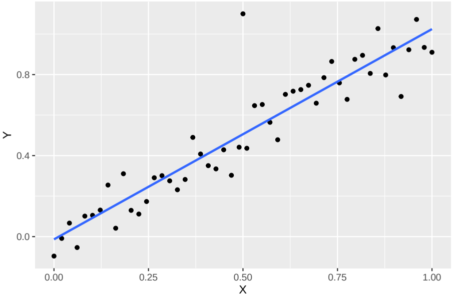
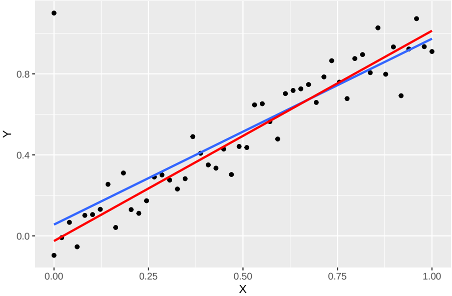


Reduce R^2 and increase RSE estimates

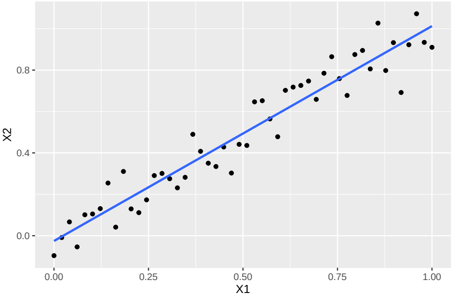
High Leverage points

Outliers which have extreme values of predictors and response are called high-leverage points



Collinearity

Collinearity occurs when predictors are highly correlated



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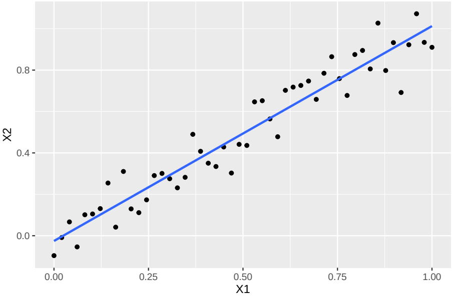


Collinearity produces high variance in estimates for β.

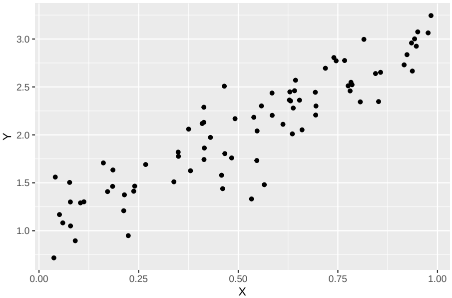
A Valid Model

Let's begin by creating a valid linear model to use as a baseline:

$$Y = 1 + 2X + \epsilon \qquad \epsilon \sim N(0, 0.25)$$

```
set.seed(700)
X <- runif(80, 0, 1)
e <- rnorm(80, 0, .25)
Y <- 1 + 2*X + e
my_data <- data.frame(X,Y)

ggplot(my_data, aes(x = X , y = Y)) + geom_point()
```

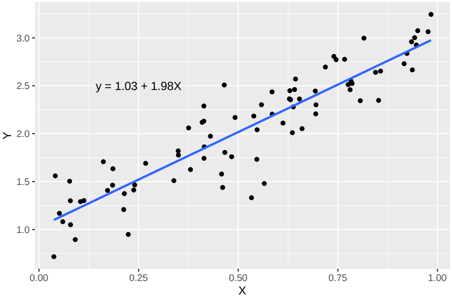


Linear Model

```
my_mod<-lm(Y ~ X, data = my_data)
beta_0 <- summary(my_mod)$coefficients[1]
beta_1 <- summary(my_mod)$coefficients[2]
c(beta_0, beta_1)
```

```
## [1] 1.025947 1.981375
```

```
ggplot(my_data, aes(x = X , y = Y)) + geom_point() + geom_smooth(method = "lm", se = F) +
  annotate(geom= "text", x = .25, y = 2.5, label = "y = 1.03 + 1.98X")
```



Model Diagnostics

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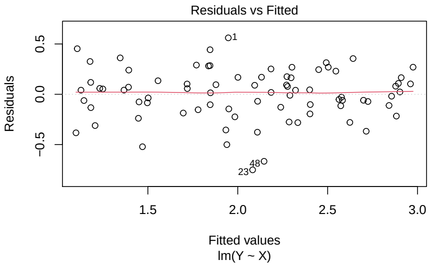
For simplicity, we'll default to the `plot` function.

Residual Plot

```
plot(my_mod, 1)
```

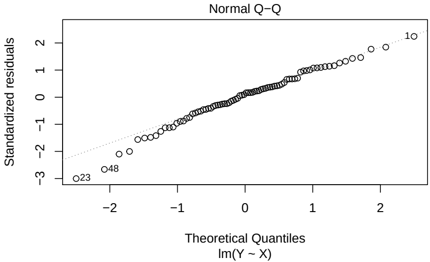


What is represented along the horizontal axis? Why?

What should we look for?

QQ Plot (Don't cry)

```
plot(my_mod, 2)
```

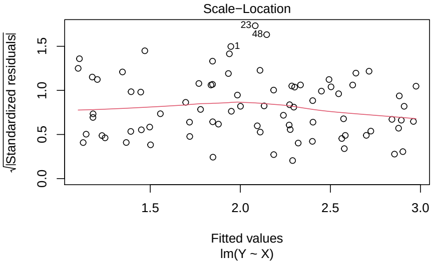


What is represented along the horizontal and vertical axes? Why?

What should we look for?

Scale-Location Plot

```r
plot(my_mod, 3)
```

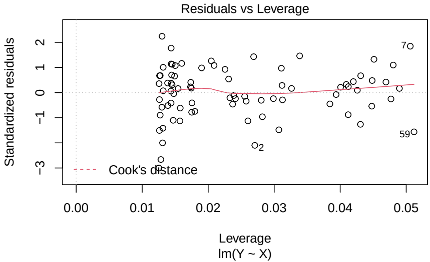


What is represented along the vertical axes? Why?

What should we look for?

Leverage Plot

```
plot(my_mod, 5)
```

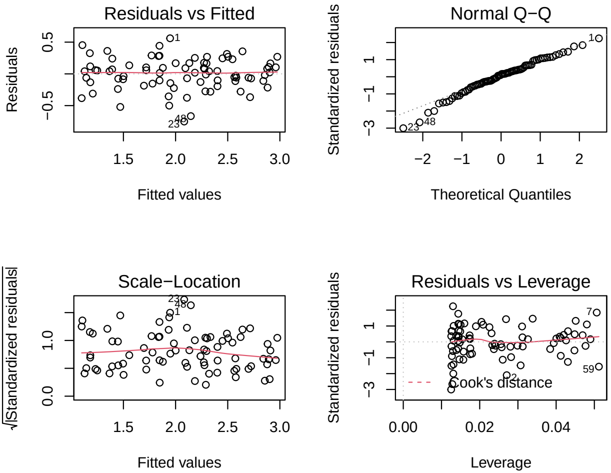


What is represented along the horizontal and vertical axes? Why?

What should we look for?

Plot Quartet

```
par(mfrow = c(2,2))
plot(my_mod)
```



Now Let's Break Things!